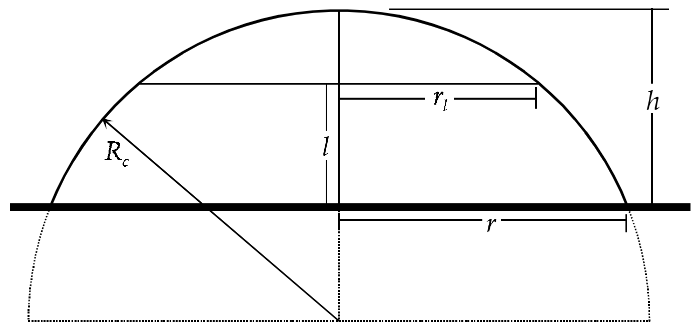
SPHERICAL DOME FORMULAS



Spheroid Dome

Circumference of base: $C = 2\pi r$

Floor Area: $F_a = \pi r^2$

Radius of Curvature: $R_c = \dfrac{r^2 + h^2}{2h}$

Surface Area: $S_a = 2\pi h R_c = \pi(h^2 + r^2)$

Radius at second level: $r_l = \sqrt{R_c^2 - (R_c - h + l)^2}$

Volume: $V_s = \frac{1}{3}\pi h^2(3R_c - h) = \frac{1}{6}\pi h(3r^2 + h^2)$

Skin Tension: $T_s = \frac{P_a R_c}{2}$

Air Pressure: $P_a = 1''$ water column $= 0.0361\,psi = 5.2\,psf$

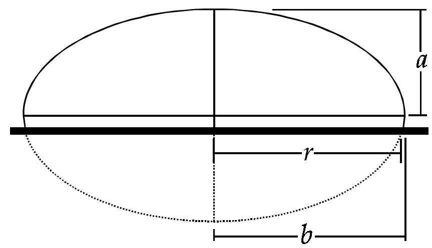
Explanation of terms

- π — is the number Pi (pronounced "pie"). Pi is the distance around the edge of a circle divided by its diameter. For our purposes the number π is a constant of 3.14159.
- d — is the diameter of the base of the dome.
- r — is the radius of the base. It is equal to half the diameter.
- R_c — is the Radius of Curvature. A spheroid dome is a segment of a sphere. Usually the top or cap of a sphere, but it can be any segment including half the sphere (hemisphere) or greater. It is helpful to think of a dome as a sliced off top of a basketball. The shape is always that of the whole basketball no matter where or how it is cut. The radius of curvature is the distance to the center of the sphere.
  Example: 40 foot diameter by 15 foot tall dome. $r = \frac{40}{2} = 20$ feet
  $R_c = \frac{r^2+h^2}{2h} = \frac{20^2+15^2}{2*15} = \frac{20*20+15*15}{30} = \frac{400+225}{30} = \frac{625}{30} = 20.83$ feet
- l — is the distance from the base of the dome to a second level (like a second floor).
- r_l — is the radius of the dome at a second level (l high). This radius is helpful to create a second floor layout or to check clearances for doors and windows. Floor area and circumference at this level is calculated using the same formulas for the whole dome (where r_l is substituted for r).
- C — is the circumference or perimiter of the base of the dome (the distance around the dome).
  Example: 40' x 15' dome — $C = \pi d = 3.14159 * 40 = 125.66$ feet
- F_a — is the area of the floor of the dome.
  Example: 40' x 15' dome — $F_a = \pi r^2 = 3.14159 * 20^2 = 3.14159 * 20 * 20 = 1,256$ ft^2
- S_a — is the surface area of the dome. (This is the equation where R_c is used most often)
  Example: 40' x 15' dome — $S_a = 2\pi h R_c = 2 * 3.14159 * 15 * 20.83 = 1,963$ ft^2

ELLIPSOID DOME FORMULAS

Ellipsoids are difficult to calculate and understand, however, they make very useful dome shapes. Our most common shape is the *oblate ellipsoid.* It looks like a standard spherical dome with a circular base, but it is "squashed" a little. The sides are more vertical and the top is flatter. This makes smaller "house" size domes that have a little more headroom along the dome wall. A *prolate ellipsoid* looks more like a watermelon. It is useful in creating a unique building shape.

Ellipse: (Let *a* be the semi-major axis and *b* be the semi-minor axis.)



Elliptical formula: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

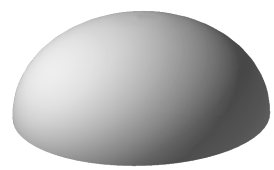
Eccentricity: $\in = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \frac{b^2}{a^2}}$

Oblate Ellipsoid: An *oblate ellipsoid* is formed by the rotation of an ellipse about its minor axis. Let *a* be the semi-major axis and *b* be the semi-minor axis. Let ∈ be the eccentricity of the revolving ellipse.

Minimum Semi-minor to Semi-major axis ratio: 1 : 1.35

Surface area for entire *oblate ellipsoid*: $S_o = \pi a^2 + \frac{\pi b^2}{2 \in} \ln\left(\frac{1 + \in}{1 - \in}\right)$

Volume for entire *oblate ellipsoid:* $V_o = \frac{4}{3}\pi b a^2$



Prolate Ellipsoid: A *prolate ellipsoid* is formed by the rotation of an ellipse about its major axis. Let *a* be the semi-major axis and *b* be the semi-minor axis. Let ∈ be the eccentricity of the revolving ellipse.

Surface area for the entire *prolate ellipsoid:* $S_p = 2\pi b^2 + \frac{2\pi ab \arcsin(\in)}{\in}$

Volume for entire *prolate ellipsoid:* $V_p = \frac{4}{3}\pi a b^2$